Fundamentals of Radio Interferometry

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Topics

- **Introduction:**
  - Why Interferometry?
  - Sensors (aka Antennas)
- **The Basic Interferometer**
  - Simplifying Assumptions
  - 'Fringe' patterns
  - Sine and Cosine Fringes
  - Response to Extended Emission
  - The Complex Correlator
- **The Real Interferometer**
  - Wide bandwidths
  - Moving Platforms/Fringe Tracking
  - Time Averaging

Why Interferometry?

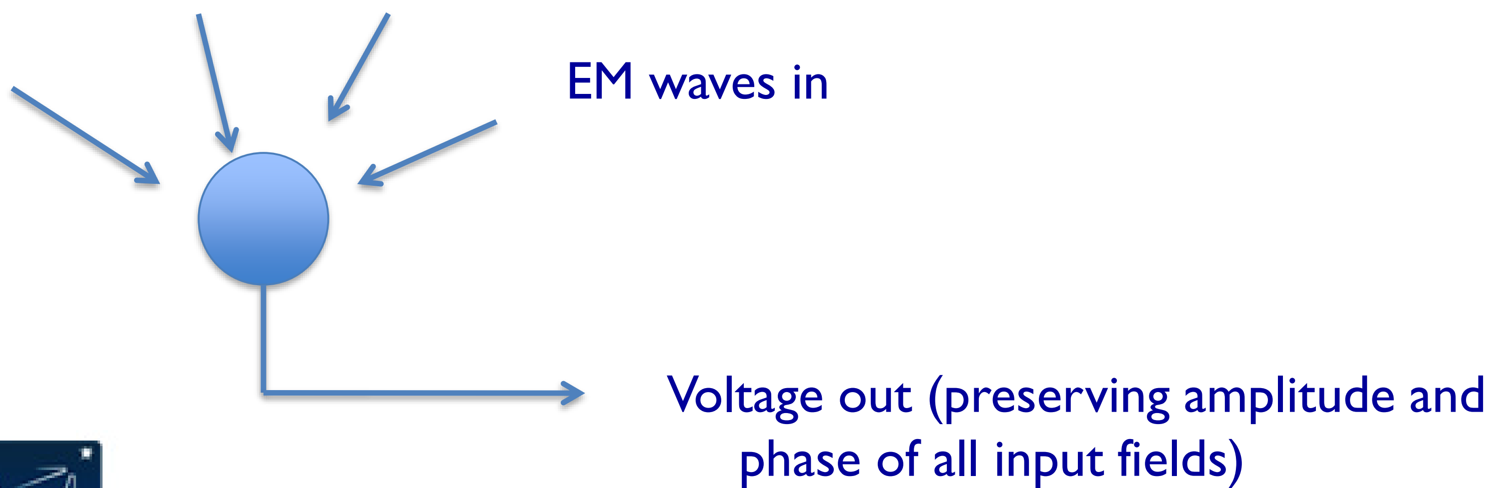
- It's all about **Diffraction** – a consequence of the wave nature of light.
- Radio telescopes coherently sum electric fields over an aperture of size D. For this, diffraction theory applies – the angular resolution is:

$$\theta_{rad} \approx \lambda / D \qquad \text{Or, in practical units} \qquad \theta_{\mathrm{arcsec}} \approx 2\,\lambda_{\mathrm{cm}} / D_{\mathrm{km}}$$

- To obtain 1 arcsecond resolution at a wavelength of 21 cm, we require an aperture of ~42 km!
- The (currently) largest single, fully-steerable aperture is the 100-m antennas in Bonn, and Green Bank. Nowhere big enough.
- Can we synthesize an aperture of that size with pairs of antennas?
- The technique of synthesizing a larger aperture through combinations of separated pairs of antennas is called 'aperture synthesis'.

The Role of the Sensor

- Coherent interferometry is based on the ability to correlate the electric fields measured at spatially separated locations.
- To do this (without mirrors) requires conversion of the electric field E(**r**,ν,t) at some place (**r**) to a voltage V(ν,t) which can be conveyed to a central location for processing.
- For our purpose, the sensor (a.k.a. 'antenna') is simply a device which senses the electric field at some place and converts this to a voltage which faithfully retains the amplitudes and phases of the electric fields.



Quasi-Monochromatic Radiation

- Analysis is simplest if the fields are perfectly monochromatic.
- This is not possible – a perfectly monochromatic electric field would both have no power ($\Delta\nu = 0$), and would last forever!
- So we consider instead 'quasi-monochromatic' radiation, where the bandwidth $d\nu$ is finite, but very small compared to the frequency: $d\nu << \nu$
- Consider then the electric fields from a small sold angle $d\Omega$ about some direction **s**, within some small bandwidth $d\nu$, at frequency ν.
- We can write the temporal dependence of this field as:

$$E_\upsilon(t) = E\cos(2\pi\upsilon t + \phi)$$

- The amplitude and phase remains unchanged to a time duration of order $dt \sim 1/d\nu$, after which new values of **E** and ϕ are needed.

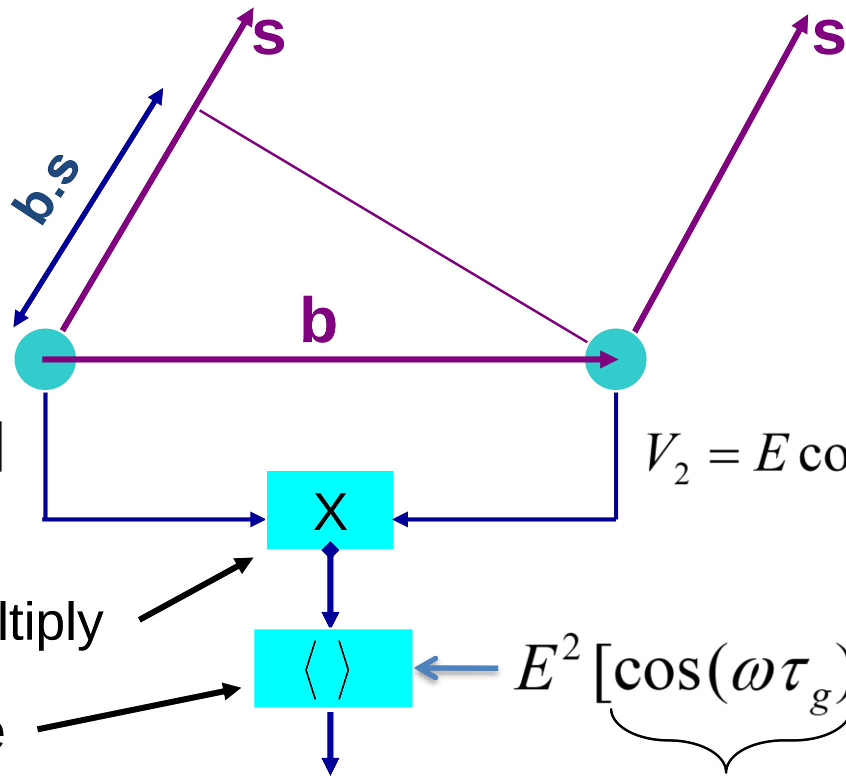
Simplifying Assumptions

- We now consider the most basic interferometer, and seek a relation between the characteristics of the product of the voltages from two separated antennas and the distribution of the brightness of the originating source emission.
- To establish the basic relations, the following simplifications are introduced:
  - Fixed in space – no rotation or motion
  - Quasi-monochromatic
  - No frequency conversions (an ‘RF interferometer’)
  - Single polarization
  - No propagation distortions (no ionosphere, atmosphere …)
  - Idealized electronics (perfectly linear, perfectly uniform in frequency and direction, perfectly identical for both elements, no added noise, …)

The Stationary, Quasi-Monochromatic Radio-Frequency Interferometer

Geometric Time Delay

$$\tau_g = \mathbf{b} \cdot \mathbf{s} / c$$



$$V_1 = E\cos[\omega(t - \tau_g)]$$

$$V_2 = E\cos(\omega t)$$

The path lengths from sensors to multiplier are assumed equal!

multiply

average

$$E^2[\cos(\omega\tau_g) + \cos(2\omega t - \omega\tau_g)]$$

Unchanging: $\cos(\omega\tau_g)$

Rapidly varying, : $\cos(2\omega t - \omega\tau_g)$

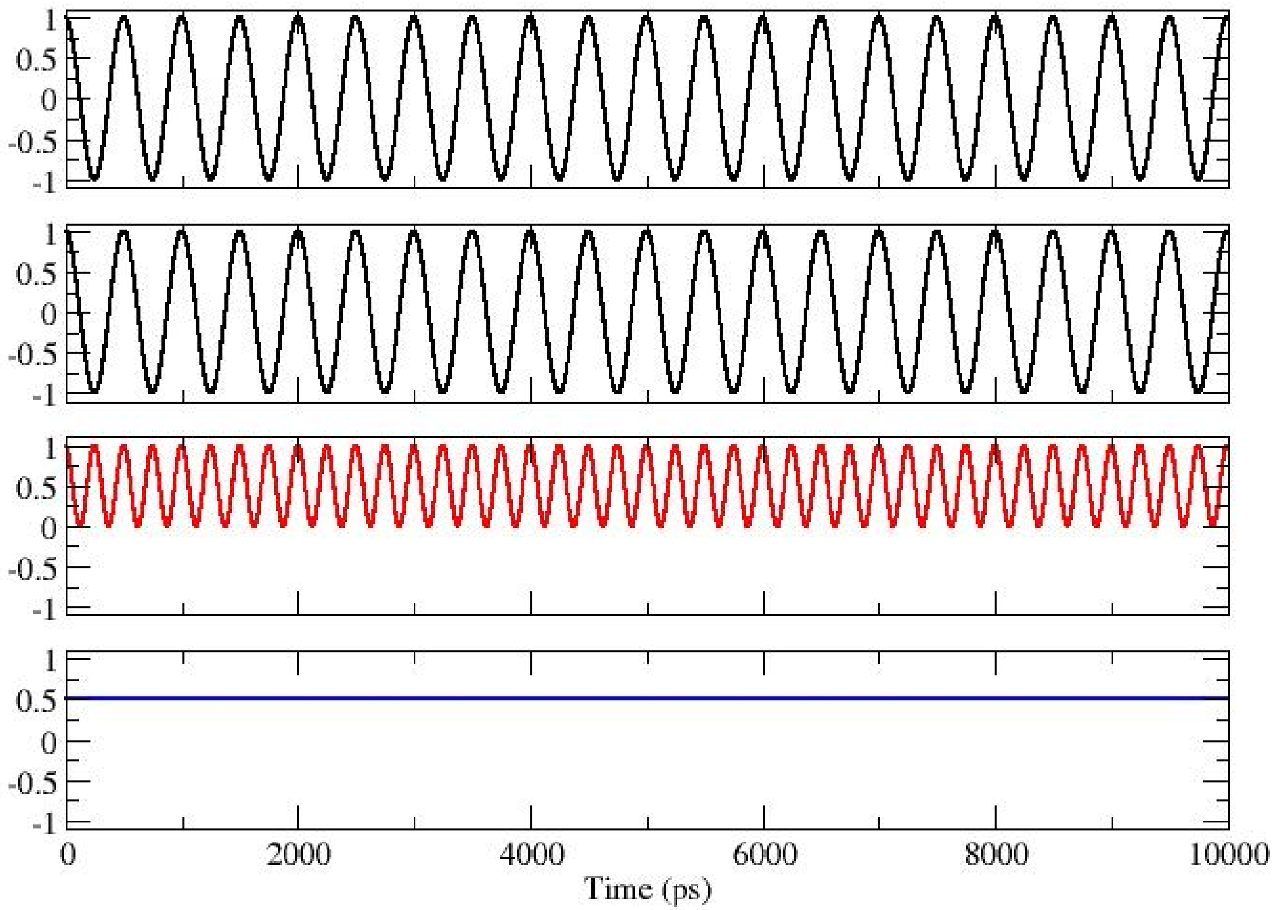
$$R_C = E^2\cos(\omega\tau_g)$$

Pictorial Example: Signals In Phase

2 GHz Frequency, with voltages in phase:

b.s = nλ, or $\tau_g = n/\nu$

- Antenna 1 Voltage
- Antenna 2 Voltage
- Product Voltage
- Average

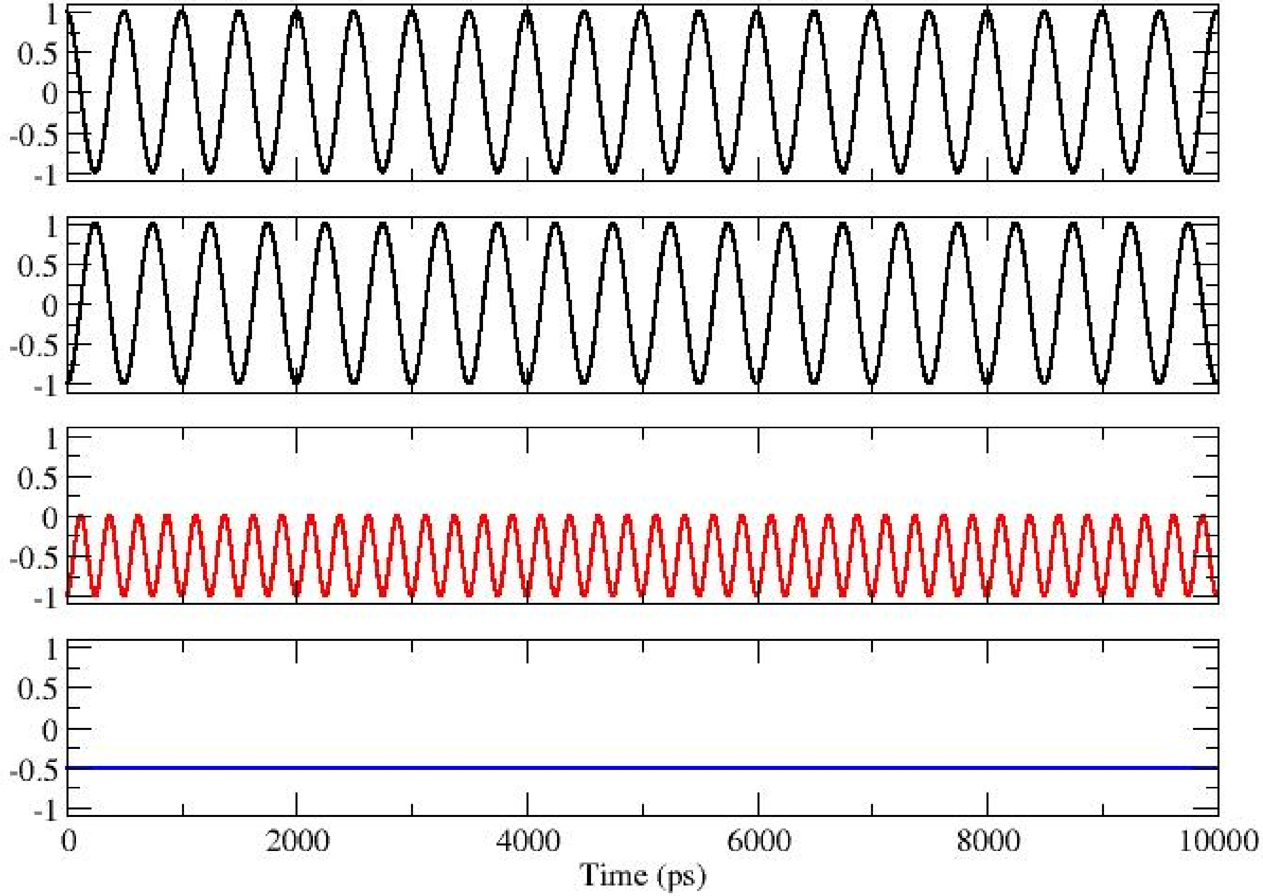


Pictorial Example: Signals out of Phase

2 GHz Frequency, with voltages out of phase:

$b.s=(n \pm \frac{1}{2})\lambda \qquad \tau_g = (2n \pm 1)/2\nu$

- Antenna 1 Voltage
- Antenna 2 Voltage
- Product Voltage
- Average

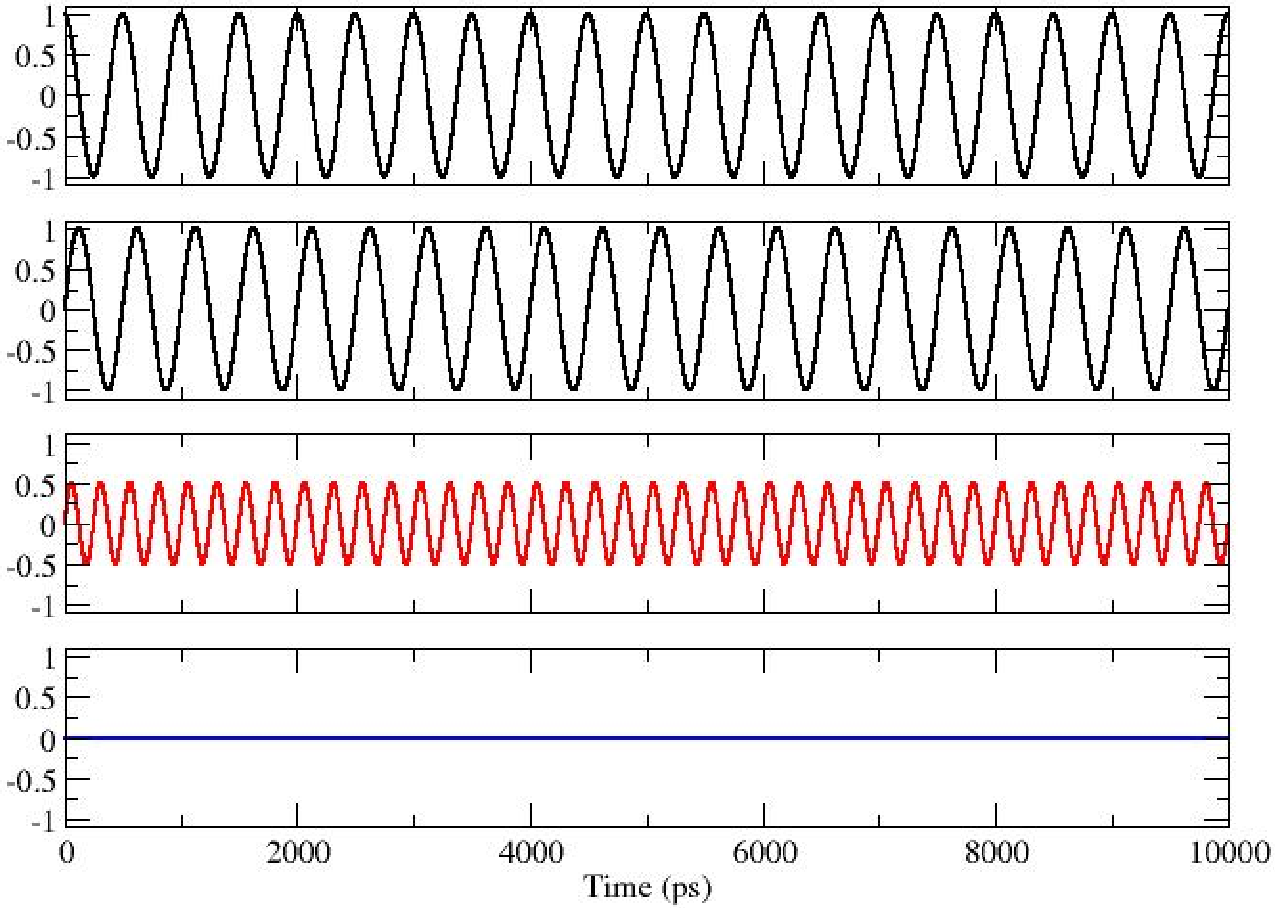


Pictorial Example: Signals in Quad Phase

2 GHz Frequency, with voltages in quadrature phase:

$b.s=(n \pm 1/4)\lambda$, $\tau_g = (4n \pm 1)/4\nu$

- Antenna 1 Voltage
- Antenna 2 Voltage
- Product Voltage
- Average



Some General Comments

- The averaged product R_C is dependent on the received power, $P = E^2/2$ and geometric delay, τ_g, and hence on the baseline orientation and source direction:

$$R_C = P\cos(\omega\tau_g) = P\cos\left(2\pi\frac{\mathbf{b}\cdot\mathbf{s}}{\lambda}\right)$$

- Note that R_C is not a function of:
  - The time of the observation -- provided the source itself is not variable!
  - The location of the baseline -- provided the emission is in the far-field.
  - The actual phase of the incoming signal – the distance of the source -- provided the source is in the far-field.
  - The frequency of the incoming signal. Indeed, the process of averaging `washes out' the oscillations.

- The strength of the product is dependent on the antenna sizes and electronic gains – but these factors can be calibrated for.

Pictorial Illustrations

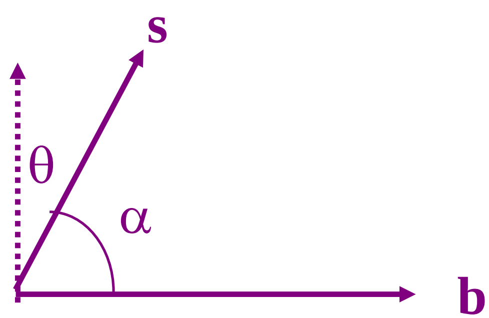
- To illustrate the response, expand the dot product in one dimension:

$$\frac{\mathbf{b} \bullet \mathbf{s}}{\lambda} = u\cos\alpha = u\sin\theta = ul$$

- Thus, the 'cosine' response can now be written

$$R_C = P\cos(\omega\tau_g) = P\cos\left(2\pi\frac{\mathbf{b}\cdot\mathbf{s}}{\lambda}\right) = P\cos(2\pi ul)$$

- Here, **u = b/λ is the baseline length in wavelengths**, and θ is the angle w.r.t. the plane perpendicular to the baseline.
- And $l = \cos\alpha = \sin\theta$ is the **direction cosine**



- So, what do these patterns 'look like' on the sky?

The 'Cosine' Interferometer Response

- Consider the response R_c, as a function of angle, for two different baselines with u = 10, and u = 25 wavelengths, to a 'unit' source. Since

$$R_C = \cos(2\pi u l)$$

- We have (for u = 10)

$$R_C = \cos(20\pi l)$$

- And (for u = 25)

$$R_C = \cos(50\pi l)$$

- These are simple functions of angle on the sky.
- Some illustrations should help.

Whole-Sky Response

- Top:

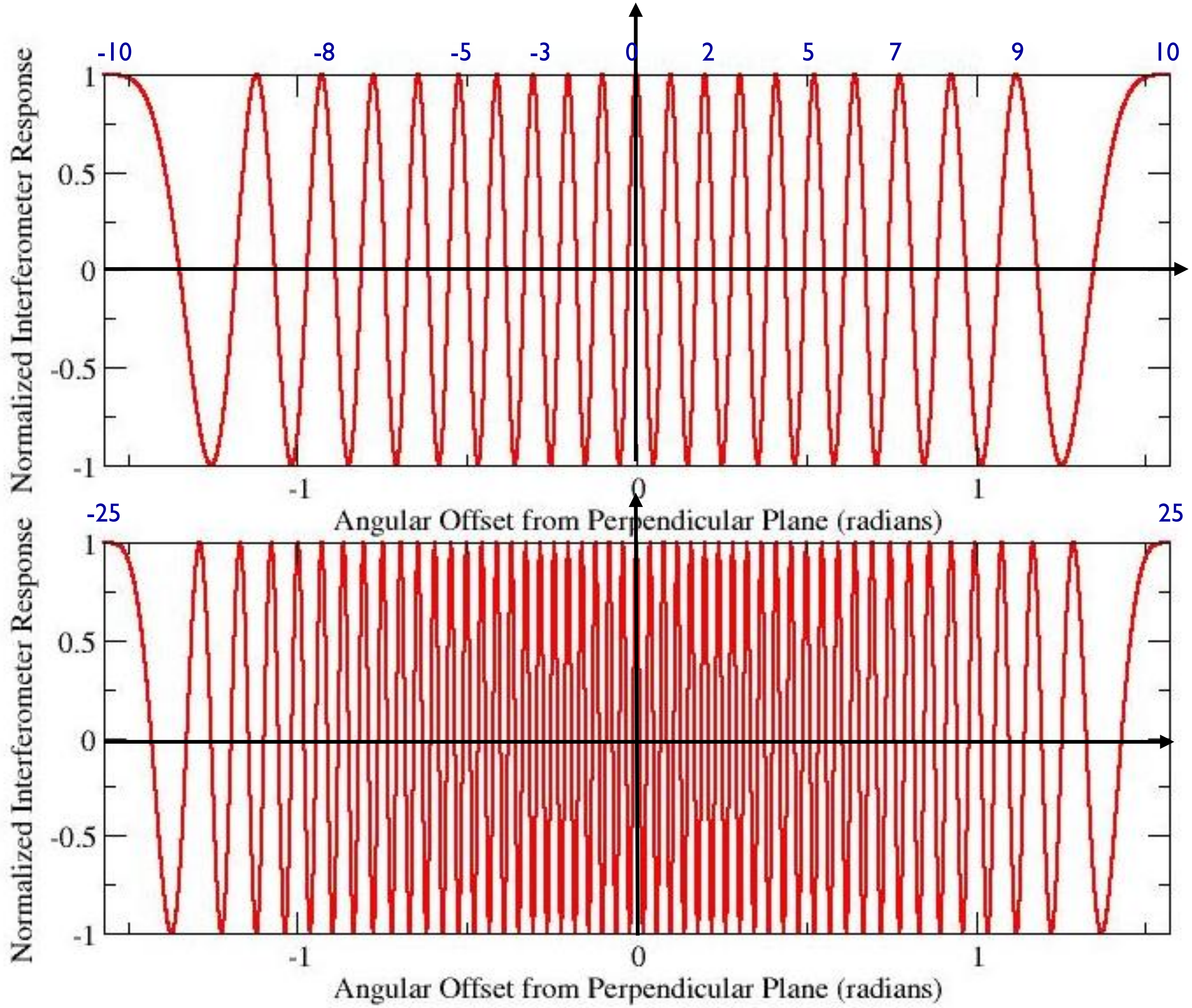
  u = 10

  There are 21 fringe maxima, and 20 fringe minima over the hemisphere.

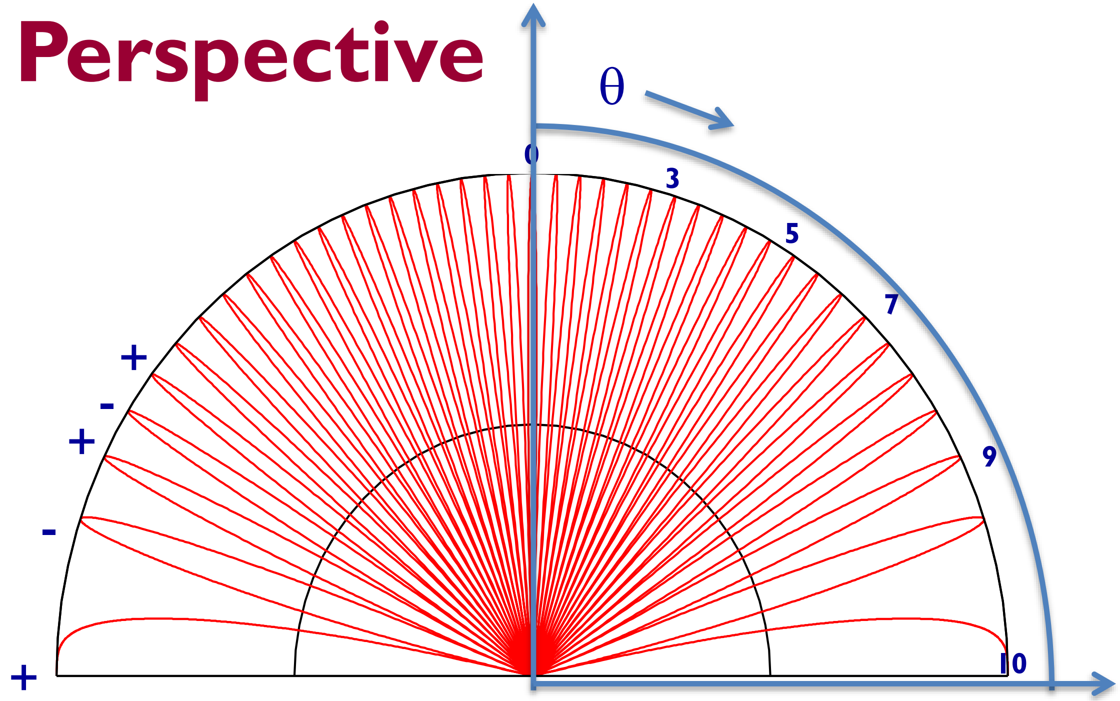
- Bottom:

  u = 25

  There are 51 fringe maxima over the hemisphere



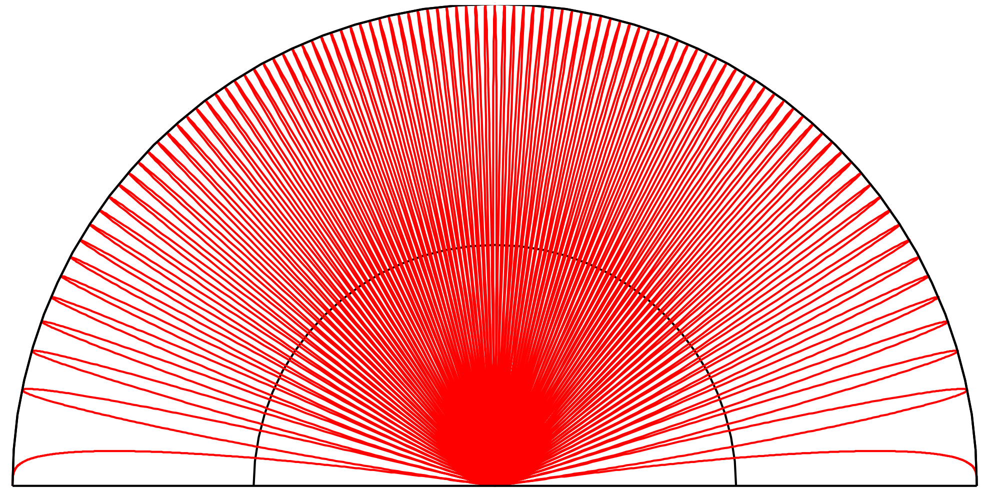
From an Angular Perspective



Top Panel:

The absolute value of the response for u = 10, as a function of angle.

The 'lobes' of the response pattern alternate in sign.



Bottom Panel:

The same, but for u = 25.

Angular separation between lobes (of the same sign) is

$\delta\theta \sim 1/u = \lambda/b$ radians.

Hemispheric Pattern

- The preceding plot is a meridional cut through the hemisphere, oriented along the baseline vector.
- In the two-dimensional space, the fringe pattern consists of a series of coaxial cones, oriented along the baseline vector.
- The figure is a two-dimensional representation when u = 2.
- As viewed along the baseline vector, the fringes show a 'bulls-eye' pattern – concentric circles.

The Effect of the Sensor

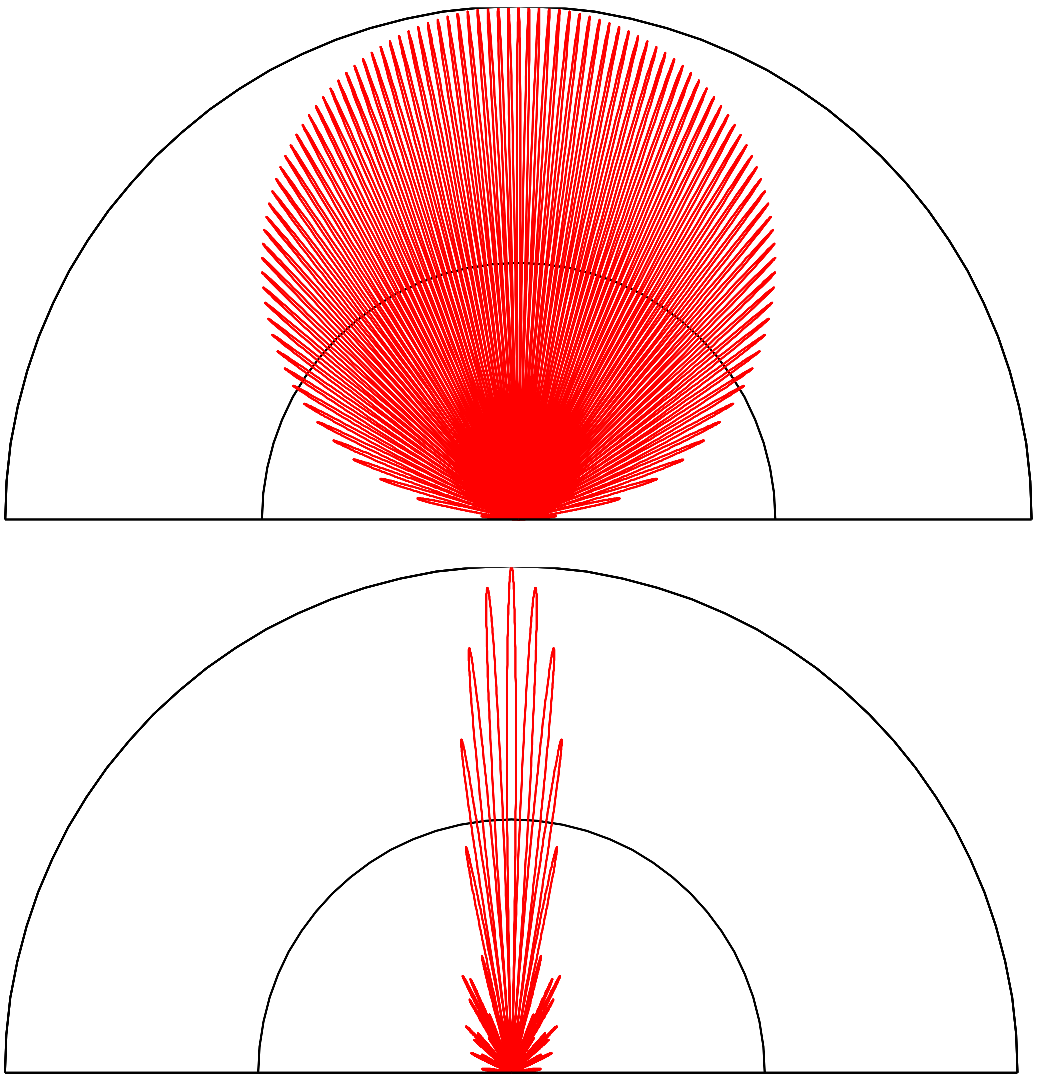
- The patterns shown presume the sensor has isotropic response.
- This is a convenient assumption, but (sadly, in some cases) doesn't represent reality.
- Real sensors impose their own patterns, which modulate the amplitude and phase of the output.
- Large sensors (a.k.a. 'antennas') have very high directivity -- very useful for some applications.

The Effect of Sensor Patterns

- Sensors (or antennas) are not isotropic, and have their own responses – both amplitude and phase.

- **Top Panel:** The interferometer pattern with a cos(θ)-like sensor response.

- **Bottom Panel:** A multiple-wavelength aperture antenna has a narrow beam, but also sidelobes.



The Response from an Extended Source

- The response from an extended source is obtained by summing the responses from each antenna to all the emission over the sky, multiplying the two, and averaging:

$$R_C = \left\langle \iint E_1 d\Omega \times \iint E_2 d\Omega \right\rangle$$

- It can then be shown, **providing the emission is spatially incoherent**, that the response to extended emission is:

$$R_C = \iint I_\nu(\mathbf{s}) \cos(2\pi \mathbf{b} \cdot \mathbf{s}/\lambda) d\Omega$$

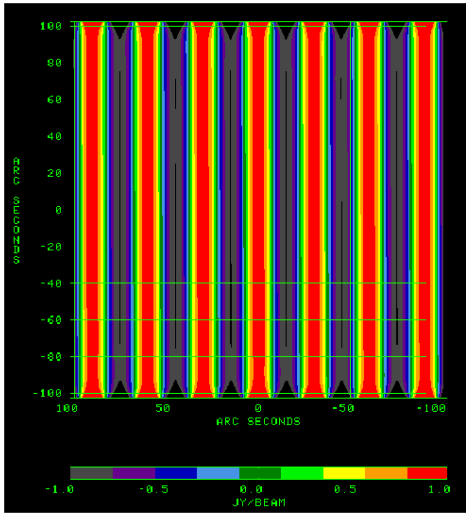
- The response is the spatial sum (integral) of the brightness distribution modulated by the cosinusoidal interferometer pattern.
- This expression links what we want: the brightness on the sky, $I_\nu(\mathbf{s})$, to something we can measure - R_C, the interferometer response.
- Can we recover $I_\nu(\mathbf{s})$ from observations of R_C?

A picture is worth 1000 words …

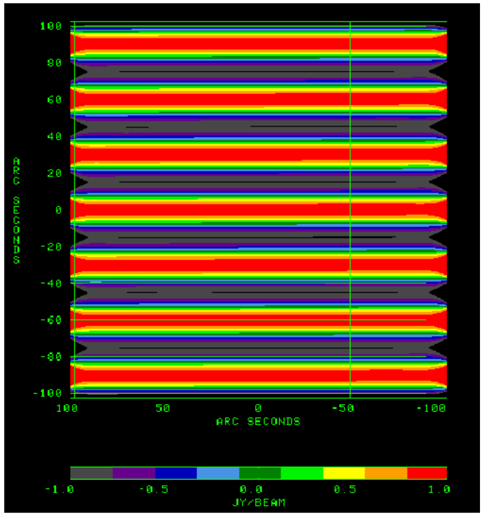
- As stated earlier, these concepts are not difficult, but are unfamiliar. We need to think in new ways, to get a deeper understanding of how all this works.
- As an aid, I have generated images of interferometer fringes, of various baseline lengths and orientations.
- I then 'observe' a real source (Cygnus A, of course), to show what the interferometer actually measures.
- For all these, the 'observations' are made at 2052 MHz. The Cygnus A image is take from real VLA data.
- To keep things simple, all simulations are done at meridian transit.

‘Real’ Fringes … 1Km Baseline at 2052 MHz

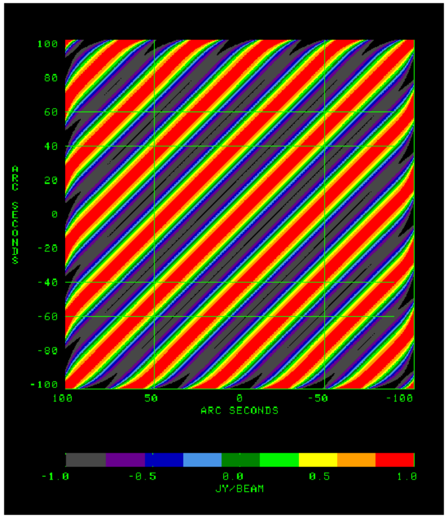
- The fringe separation given by baseline length in wavelengths, the orientation given by the orientation of the baseline.



East-West baseline makes vertical fringes



North-South baseline makes horizontal fringes



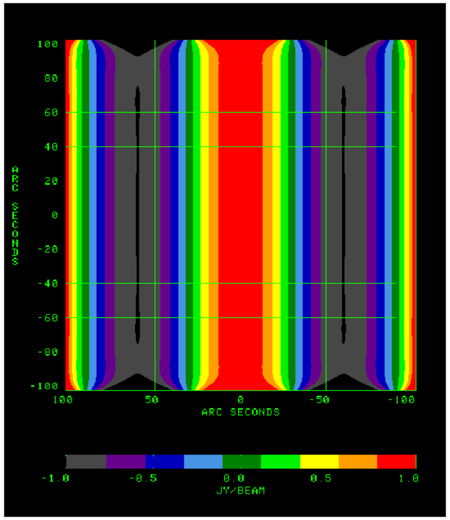
Rotated baseline makes rotated fringes

- Fringe angular spacing given by baseline length in wavelengths:

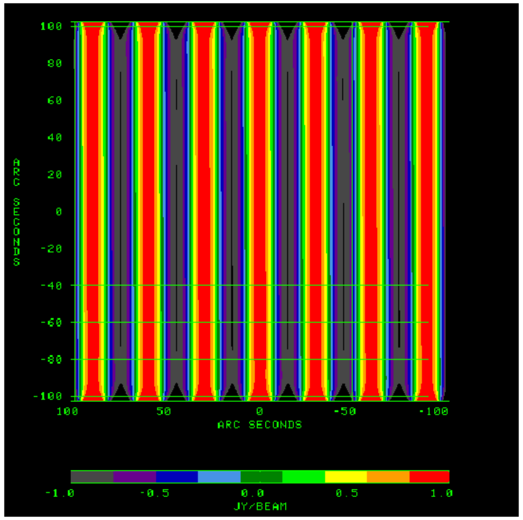
$$\Delta\theta = \lambda / B = 30.2''$$

Longer Baselines => Smaller Fringes

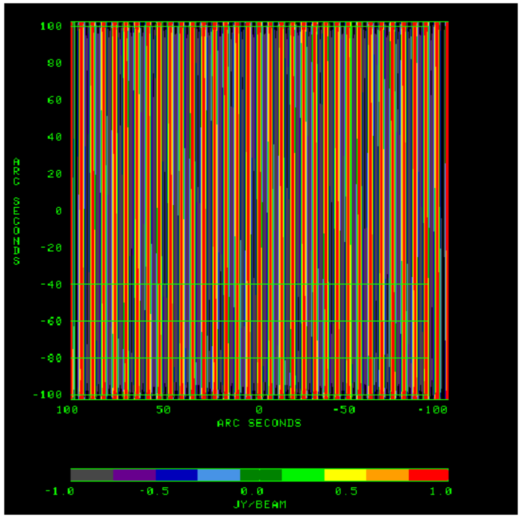
- Longer baselines generate finer fringes:



250 meter baseline
120 arcsecond fringe



1000 meter baseline
30 arcsecond fringe

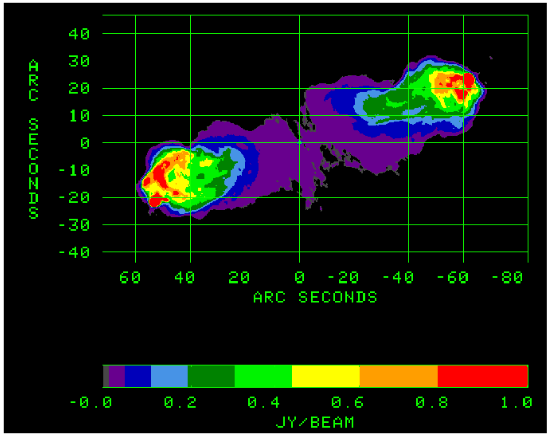


5000 meter baseline
6 arcsecond fringe

- What the interferometer measures is the integral (sum) of the product of these pattern with the actual brightness.

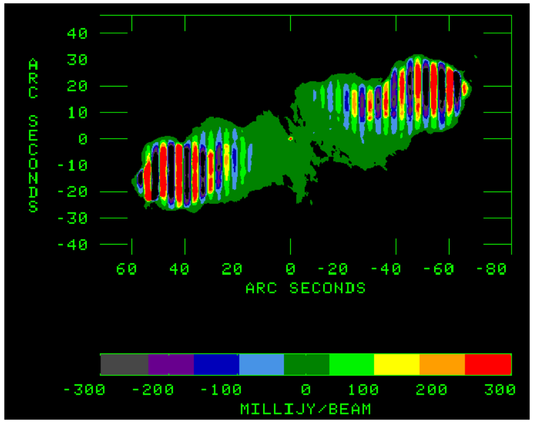
For a Real Source (Cygnus A = 3C405)

- Cygnus A is a powerful, nearby radio galaxy.
- The left panel shows the actual brightness.
- The other two panels show how the 5km-baseline (6 arcsecond fringe spacing) interferometer 'sees' it



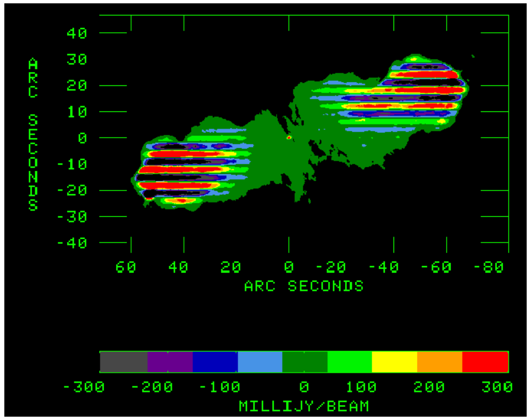
Zero-Spacing Image

Sum = 999 Jy



5 km EW spacing

Sum = 61 Jy



5 km NS spacing

Sum = -16 Jy

Some Points to Ponder …

- If the target source is a 'point source', the interferometer response is the same for every baseline.
  - 'Point Source' is an object much much smaller than the fringe spacing.
- The interferometer response to a real (Stokes I – all positive) source can be negative.
  - Although the response is proportional to source power, there is no requirement that it be positive.
- As the baseline gets longer, the response goes to zero.
  - At this point, the source is said to be 'resolved out'.
- As the baseline get shorter, the response goes to the total source flux.
  - This is termed the 'zero spacing flux'.

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So … What Good is All This?

- The interferometer casts a cosinusoidal pattern on the sky, with the result that we obtain a response which is some function of the source brightness and the fringe separation and orientation.
- How does that get us to our goal of determining the actual brightness?
- Time for some mathematics…. Starting with a seeming digression about odd and even functions.
- (All will be clear shortly…)

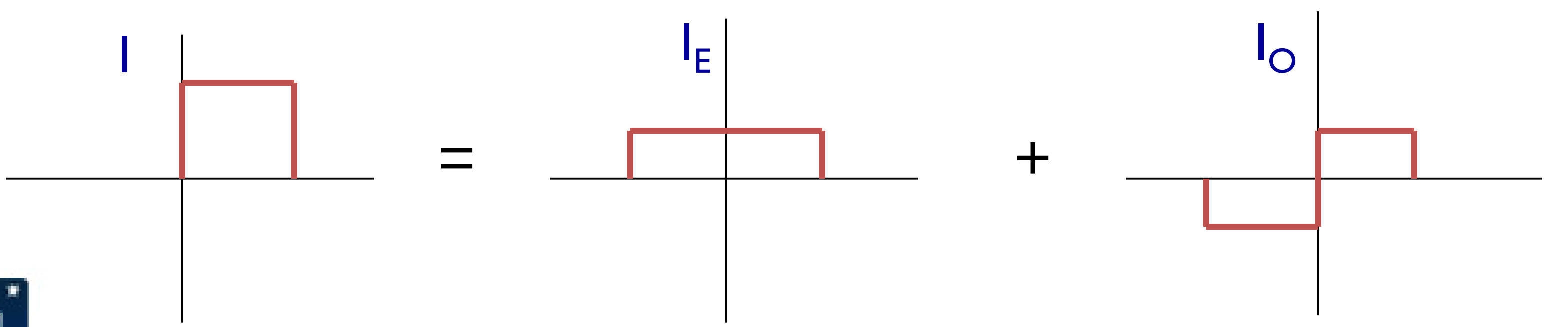
A Short Mathematics Digression – Odd and Even Functions

- Any real function, I(x,y), can be expressed as the sum of two real functions which have specific symmetries:

$$I(x,y) = I_E(x,y) + I_O(x,y)$$

An even part:
$$I_E(x,y) = \frac{I(x,y) + I(-x,-y)}{2} = I_E(-x,-y)$$

An odd part:
$$I_O(x,y) = \frac{I(x,y) - I(-x,-y)}{2} = -I_O(-x,-y)$$



The Cosine Correlator is Blind to Odd Structure

- The correlator response, R_c:

$$R_C = \iint I_\nu(\mathbf{s}) \cos(2\pi \mathbf{b} \cdot \mathbf{s} / \lambda) d\Omega$$

is not enough to recover the correct brightness. Why?

- Suppose that the source of emission has a component with odd symmetry, for which

$$I_o(\mathbf{s}) = -I_o(-\mathbf{s})$$

- Since the cosine fringe pattern is even, the response of our interferometer to the odd brightness distribution is 0!

$$R_C = \iint I_O(\mathbf{s}) \cos(2\pi \mathbf{b} \cdot \mathbf{s} / \lambda) d\Omega = 0$$

- Hence, we need more information if we are to completely recover the source brightness.

Why Two Correlations are Needed

- The integration of the cosine response, R_c, over the source brightness is sensitive to only the even part of the brightness:

$$R_C = \iint I(\mathbf{s})\cos(2\pi \mathbf{b}\cdot\mathbf{s}/\lambda)d\Omega = \iint I_E(\mathbf{s})\cos(2\pi \mathbf{b}\cdot\mathbf{s}/\lambda)d\Omega$$

since the integral of an odd function (I_O) with an even function (cos x) is zero.

- To recover the 'odd' part of the intensity, I_O, we need an 'odd' fringe pattern. Let us replace the 'cos' with 'sin' in the integral

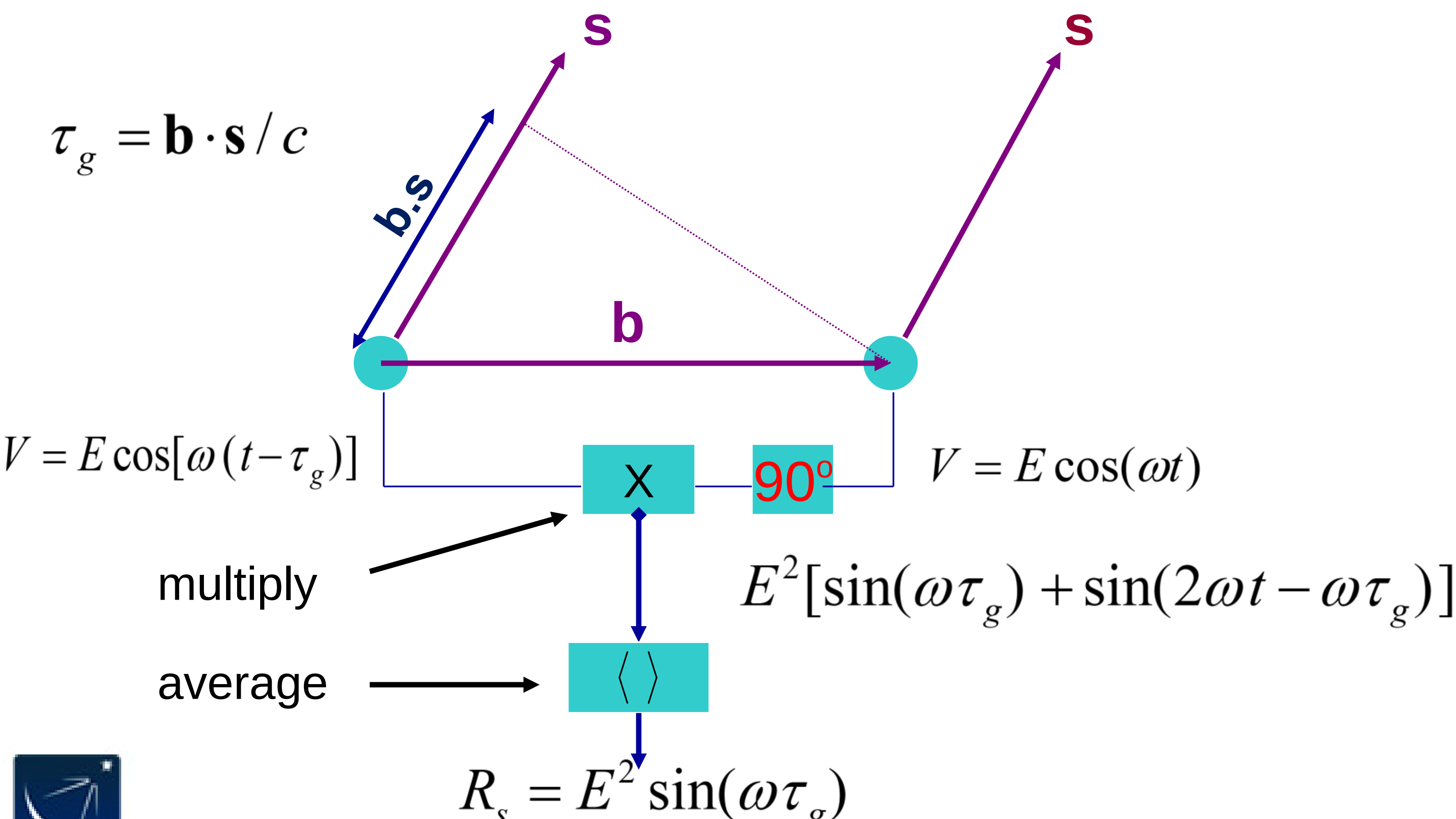
$$R_S = \iint I(\mathbf{s})\sin(2\pi \mathbf{b}\cdot\mathbf{s}/\lambda)d\Omega = \iint I_O(\mathrm{s})\sin(2\pi \mathbf{b}\cdot\mathbf{s}/\lambda)d\Omega$$

since the integral of an even times an odd function is zero.

- To obtain this necessary component, we must make a 'sine' pattern.

Making a SIN Correlator

- We generate the 'sine' pattern by inserting a 90 degree phase shift in one of the signal paths.



Define the Complex Visibility

- We now DEFINE a complex function, **the complex visibility, V,** from the two independent (real) correlator outputs R_C and R_S:

$$V = R_C - iR_S = Ae^{-i\phi}$$

where

$$A = \sqrt{R_C^2 + R_S^2}$$

$$\phi = \tan^{-1}\left(\frac{R_S}{R_C}\right)$$

- This gives us a beautiful and useful relationship between the source brightness, and the response of an interferometer:

$$V_\upsilon(\mathbf{b}) = R_C - iR_S = \iint I_\nu(s) e^{-2\pi i \mathbf{b}\cdot\mathbf{s}/\lambda} d\Omega$$

- Under some circumstances, this is a 2-D Fourier transform, giving us a well established way to recover $I(\mathbf{s})$ from $V(\mathbf{b})$.

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The Complex Correlator and Complex Notation

- A correlator which produces both 'Real' and 'Imaginary' parts – or the Cosine and Sine fringes, is called a 'Complex Correlator'
  - For a complex correlator, think of two independent sets of projected sinusoids, 90 degrees apart on the sky.
  - In our scenario, both components are necessary, because we have assumed there is no motion – the 'fringes' are fixed on the source emission, which is itself stationary.
- The complex output of the complex correlator also means we can use complex analysis throughout: Define the analytic signals:

$$\mathcal{V}_1 = Ae^{-i\omega t}$$

$$\mathcal{V}_2 = Ae^{-i\omega(t-\mathbf{b}\bullet\mathbf{s}/c)}$$

Note: The real parts of $\mathcal{V}$ are the real signals: V = Re($\mathcal{V}$)

- Then:

$$P_{corr} = \left\langle \mathcal{V}_1 \mathcal{V}_2^* \right\rangle = A^2 e^{-i\omega\,\mathbf{b}\bullet\mathbf{s}/c} = A^2 e^{-i2\pi\,\mathbf{b}\bullet\mathbf{s}/\lambda}$$

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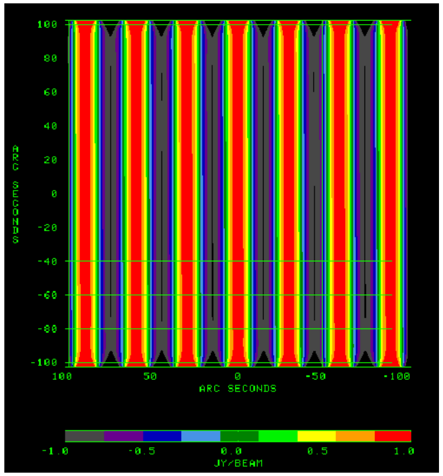
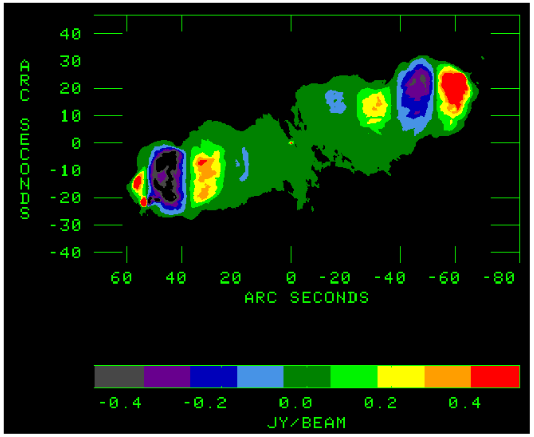
Some Pictures, to Illustrate This Point

- We now have two (real) correlators, whose patterns are phase shifted by 90 degrees on the sky:

COS



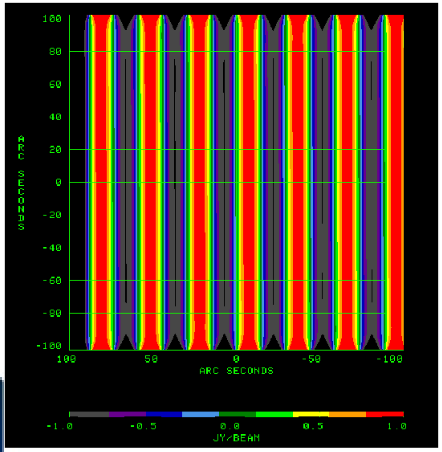


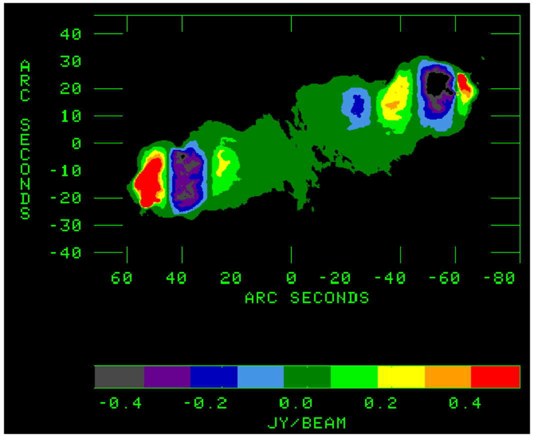
69 Jy

A=103 Jy

ϕ=48

SIN





77 Jy

More Thoughts to Ponder (at 3AM …)

- The complex visibility **amplitude** is independent of the source location, and linearly related to source flux density.
- The complex visibility **phase** is a function of source location, and independent of source flux density.
- Reversing the elements of an interferometer ('turning it around') negates the phase of the complex visibility, and leaves the amplitude unchanged.
  - For those of you familiar with Fourier transforms, the equivalent statement is that:
    - 'Since the source brightness is a real function, its Fourier transform is Hermitian'.

Some Comments on Visibilities

- The Visibility is a unique function of the source brightness.
- The two functions are related through a Fourier transform. $V_{\upsilon}(u,v) \Leftrightarrow I(l,m)$
- An interferometer, at any one time, makes one measure of the visibility, at baseline coordinate (u,v).
- `Sufficient knowledge' of the visibility function (as derived from an interferometer) will provide us a `reasonable estimate' of the source brightness.
- How many is 'sufficient', and how good is 'reasonable'?
- These simple questions do not have easy answers…

Final Comments …

- The formalism presented here presumes much … including that there is no motion between source and interferometer.
- You don't *need* a complex correlator – one can imaging a situation where the interferometer is placed on a slowly rotating platform, which 'sweeps' the fringes through the source.
- Real interferometers are on a rotating platform (the Earth), so why do we use complex correlators?
- The answer to this, and a host of other practical issues, are the subjects of my next lecture.

`Real' Interferometer Topics

- Here we relax the conditions imposed earlier.
- We will cover:
  - Real Sensors (aka 'antennas')
  - Finite bandwidth
  - Rotating reference frames (source motion)
  - Finite time averaging
  - Local Oscillators and Frequency Downconversion
- But I won't discuss polarization, sensitivity or calibration. These are topics for following lectures.

Review

- In the previous lecture, I set down the principles of Fourier synthesis imaging.
- I showed: $$V_\upsilon(\mathbf{b}) = R_C - iR_S = \iint I_\nu(s) e^{-2\pi i \mathbf{b}\cdot\mathbf{s}/\lambda} d\Omega$$

  Where the intensity I_ν is a real function, and the visibility V(**b**) is complex and Hermitian.
- The model used for the derivation was idealistic – not met in practice. We presumed:
  - Idealized Sensors (no dependence on direction)
  - Monochromatic radiation
  - Stationary reference frame.
  - RF throughout
- We now relax, in turn, these restrictions.

Real Sensors

- I noted earlier that real sensors impose their own attenuation functions $G_1(\Omega)$ and $G_2(\Omega)$ on the incoming signals.
- Presuming these are constant in angle w.r.t. the target source, the result is that the recovered image intensity is attenuated:

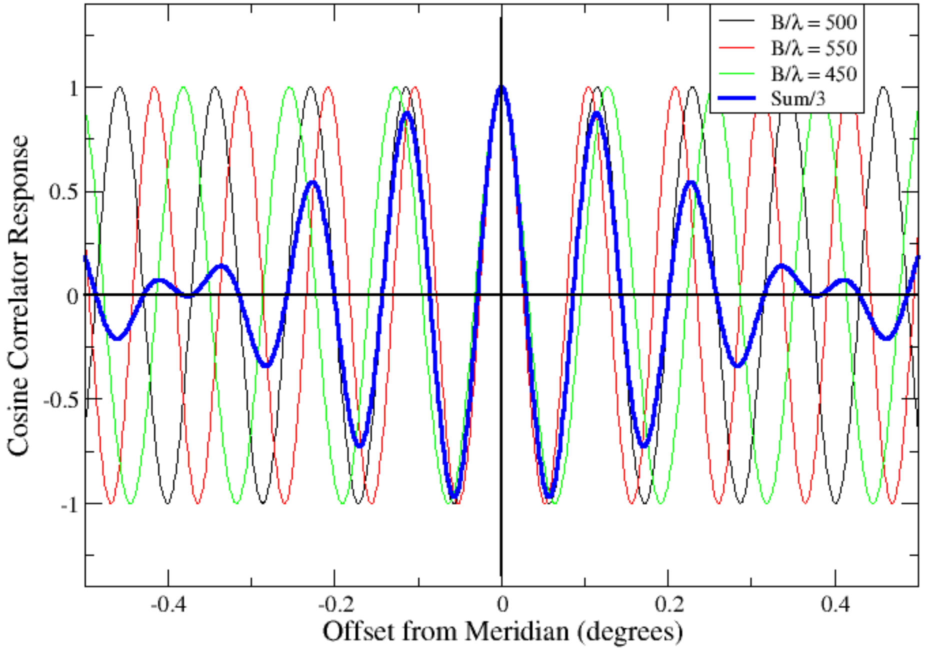
$$I_{obs} = I_{true} G_1 G_2^*$$

- There are two ways of dealing with this attenuation:
  1. Correct the image by dividing by the known beams, or
  2. Modify the measured visibilities to the un-attenuated values ('A-Projection').
- Both of these lie (well) beyond the scope of these introductory lectures.

Effect of Finite Bandwidth

- A baseline has a fixed physical length, B. But the fringe pattern depends on its length **in wavelengths**.
- Each slice of wavelength has a pattern with angular separation of ~ λ/B .
- Each component has a maximum at the n=0 fringe (meridional plane).
- They get increasingly out of step as n gets larger.

- A simple illustration – three wavelength components from the same physical baseline.
- The net result is the sum over all components.
- Here, this is shown in the thick blue line.



The Effect of Bandwidth -- Analysis.

- To find the finite-bandwidth response, we integrate our fundamental response over a frequency response G(ν), of width $\Delta\nu$, centered at ν_0

$$V = \int \left(\frac{1}{\Delta\nu} \int_{\upsilon_0-\Delta\upsilon/2}^{\upsilon_0+\Delta\upsilon/2} I(\mathbf{s},\upsilon) G_1(\upsilon) G_2^*(\upsilon) e^{-i2\pi\upsilon\tau_g} d\upsilon \right) d\Omega$$

- If the source intensity does not vary over the bandwidth, and the instrumental gain parameters G_1 and G_2 are square and identical, then

$$V = \iint I_\upsilon(\mathbf{s}) \frac{\sin(\pi\tau_g\Delta\upsilon)}{\pi\tau_g\Delta\upsilon} e^{-2i\pi\nu_0\tau_g} d\Omega = \iint I_\upsilon(\mathbf{s}) \mathrm{sinc}(\tau_g\Delta\upsilon) e^{-2i\pi\nu_0\tau_g} d\Omega$$

where the **fringe attenuation function**, sinc(x), is defined as:

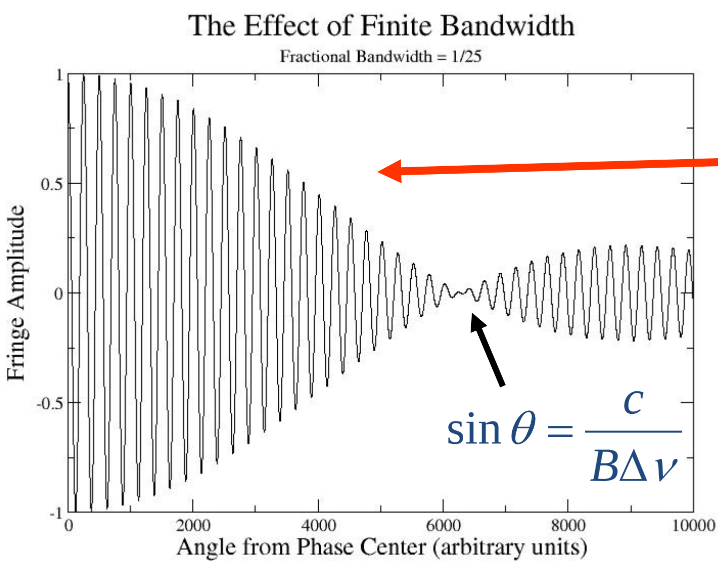
$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

Bandwidth Effect Example

- For a square bandpass, the bandwidth attenuation reaches a null when $\tau_g \Delta\nu = 1$, or

$$\sin\theta = {\lambda / B}\big/{\Delta\upsilon / \upsilon_0} = \frac{c}{B\Delta\nu}$$

- For the Jansky VLA, $\Delta\nu$ = 1 MHz, and B = 35 km, (A configuration) then the null occurs at about 30 arcminutes off the meridian.
- A 10% loss in fringe amplitude occurs at an offset of ~7.5 arcminutes.
- This is a real loss of visibility amplitude – because the effect depends on offset angle and baseline length, it effectively limits the resolution.
- This ‘chromatic aberration’ can only be reduced by reducing the bandwidth.
- Reducing bandwidth causes loss of sensitivity – so we simply add channels.



$$\mathrm{sinc}(\tau_g \Delta\upsilon) = \mathrm{sinc}\left(\frac{B\Delta\upsilon}{c}\sin\theta\right)$$

Number of fringes between peak and null:

$$N \sim \frac{c}{B\Delta\upsilon}\frac{B}{\lambda} \sim \frac{\upsilon}{\Delta\upsilon}$$

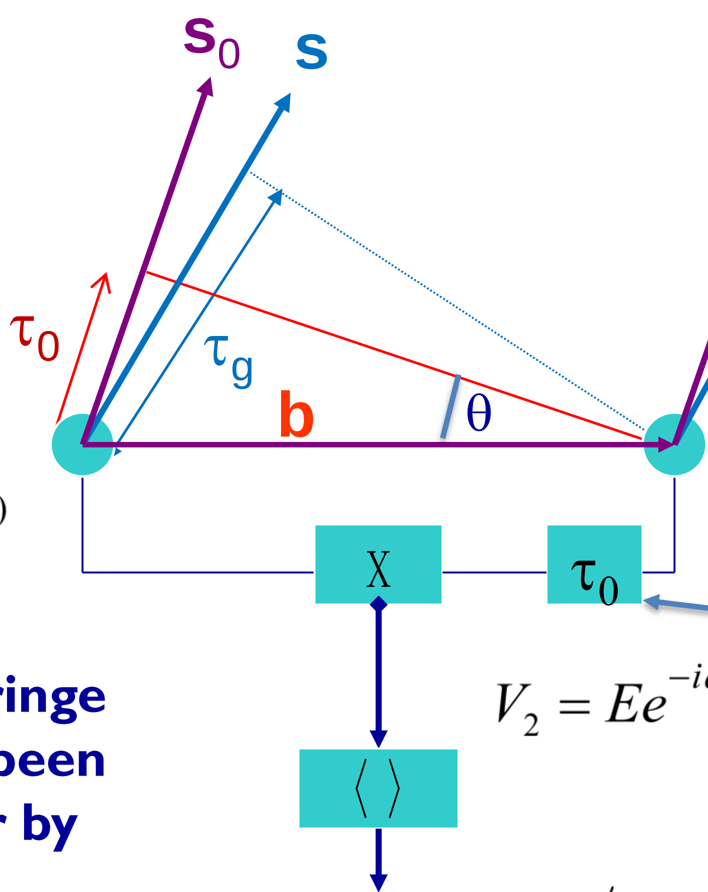
Observations off the Meridian

- In our basic scenario -- stationary source, stationary interferometer -- the effect of finite bandwidth will strongly attenuate the fringe amplitudes from sources far from the meridional plane.
- Since each baseline has its own plane, the only point on the sky free of attenuation for all baselines is a small angle around the zenith (presuming all baselines are coplanar).
- Hence, for our model interferometer, we can only observe objects within a few arcminutes of the zenith.
- Suppose we wish to observe an object far from the zenith?
- Best way is to shift the entire 'fringe packet' to the position of interest by adding time delay to the antenna closer to the source.

Adding Time Delay

$$\tau_g = \mathbf{b} \cdot \mathbf{s} / c$$

$$\tau_0 = \mathbf{b} \cdot \mathbf{s}_0 / c$$



$\mathbf{s_0}$ = reference (delay) direction

$\mathbf{s}$ = general direction vector

$$V_1 = Ee^{-i\omega(t-\tau_g)}$$

$$V_2 = Ee^{-i\omega t}$$

New box: Time delay

$$V_2 = Ee^{-i\omega(t-\tau_0)}$$

The entire fringe pattern has been shifted over by angle

$\sin\theta = c\tau_0/b$

$$V = \langle V_1 V_2^* \rangle = E^2 e^{-i[\omega(\tau_0-\tau_g)]}$$

$$= E^2 e^{i2\pi[\nu\mathbf{b}\cdot(\mathbf{s}-\mathbf{s}_0)/c]}$$

NRAO

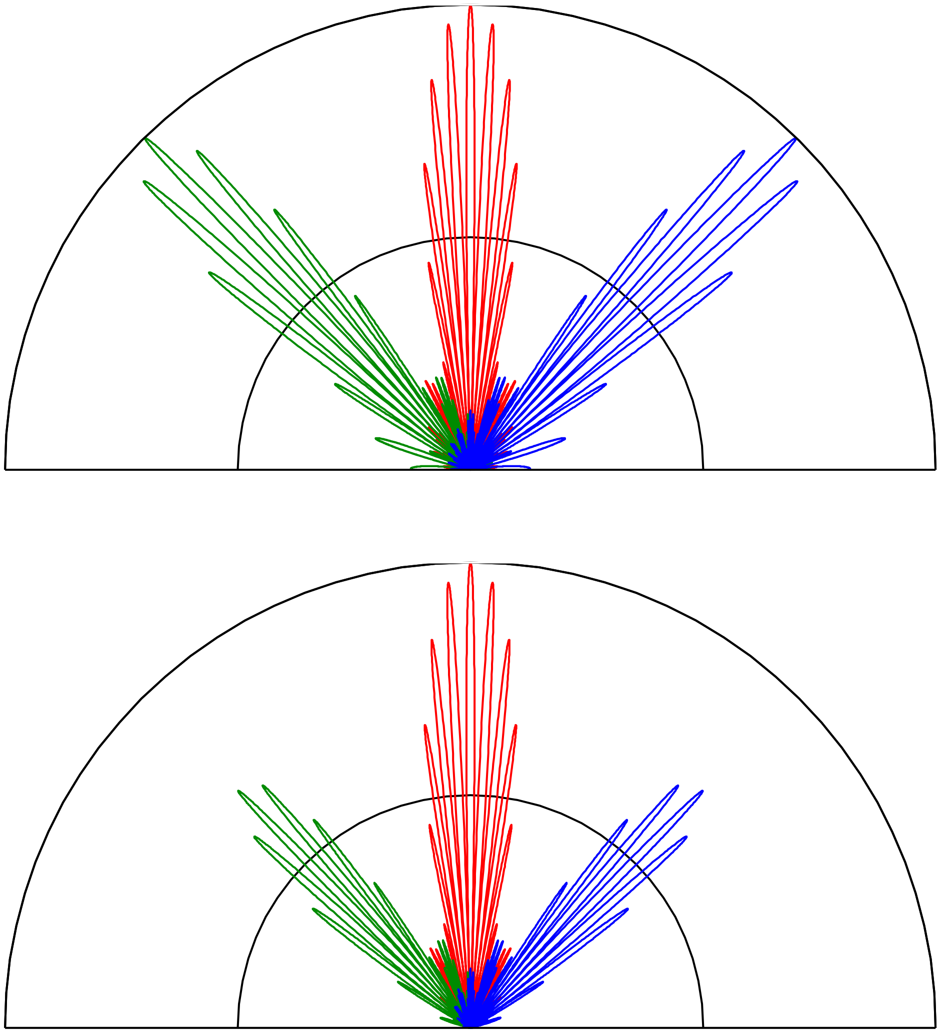
Illustrating Delay Tracking

- Top Panel:

  Delay has been added and subtracted to move the delay pattern to the source location.

- Bottom Panel:

  A cosinusoidal sensor pattern is added, to illustrate losses from a fixed sensor.



Observations from a Rotating Platform

- Real interferometers are built on the surface of the earth – a rotating platform. From the observer's perspective, sources move across the sky.
- Since we know how to adjust the interferometer timing to move its coherence pattern to the direction of interest, it is a simple step to continuously move the pattern to follow a moving source.
- All that is necessary is to continuously add time delay, with an accuracy $\delta\tau \sim 1/10\Delta\nu$ to minimize bandwidth loss.
- But there's one more issue to keep in mind, which is that of phase.

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Phase Tracking …

- Adding time delay will prevent bandwidth losses for observations off the baseline's meridian.
- Delay insertion is finite – not continuously variable.
- Between delay settings, the source is moving through the interferometer pattern – a rapidly changing phase.
- The 'natural fringe rate' – due to earth's rotation, is given by

$$\upsilon_f = u\omega_e \cos\delta \quad \text{Hz}$$

- Where u = B/λ, the (E-W) baseline in wavelengths, and ω_e =7.3x10^{-5} rad/s is the angular rotation rate of the earth.
- For a million-wavelength baseline, ν_f ~ 70 Hz – that's fast.
- If we leave things this way, we have to sample the output at at least twice this rate. A lot of data!

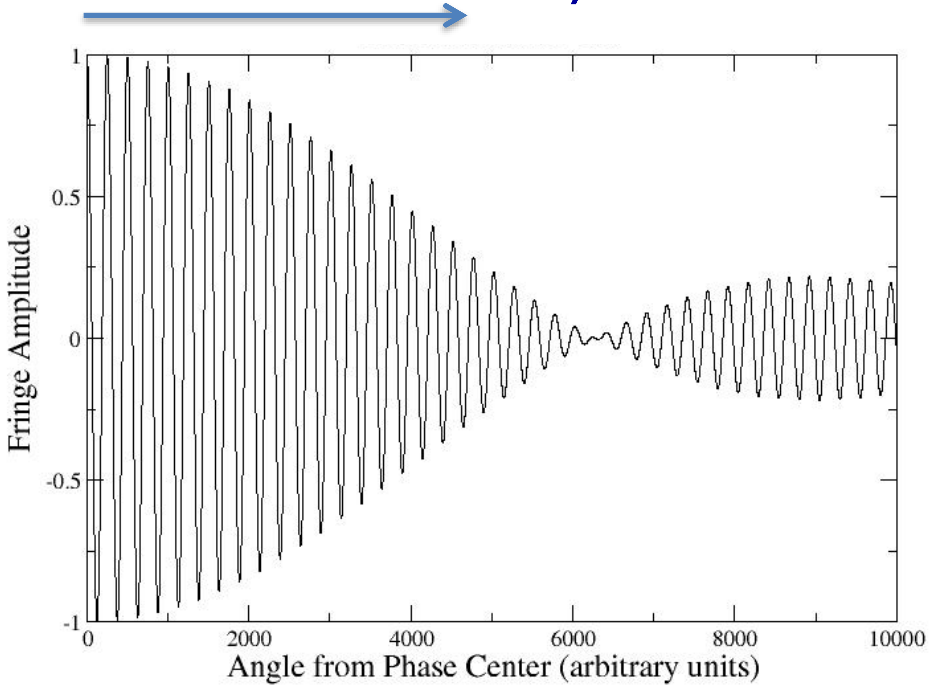
Following a Moving Object.

- There is **no useful information** in this fringe rate – it's simply a manifestation of the platform rotation.
- Tracking, or 'stopping' the fringes greatly slows down the post-correlation data processing/archiving needs.
- To 'stop' the fringes, we must adjust the phase in one path.
- How fast:
  - Tracking delay: $$\upsilon_d >> \frac{\Delta\upsilon}{\upsilon}\frac{B}{\lambda}\omega_e \cos\delta \sim 1\,\mathrm{Hz}$$
  - Tracking phase: $$\upsilon_f >> \frac{B}{\lambda}\omega_e \cos\delta \sim 70\ \mathrm{Hz}$$
- The rates given are appropriate for 35 km baselines, 128 MHz bandwidth, and 3 cm wavelength.
- For the 'RF' interferometer, delay insertion does both.

Emphasis:

- Shown again is the fringe pattern of a real wide-band baseline.
- To preserve the visibility amplitude, we must re-set delays before the source moves too far down the pattern.
- To maintain a stable phase, we must reset the phase ~$\nu/\delta\nu$ times faster.
- I'll mention later how this is done.

Source moves this way



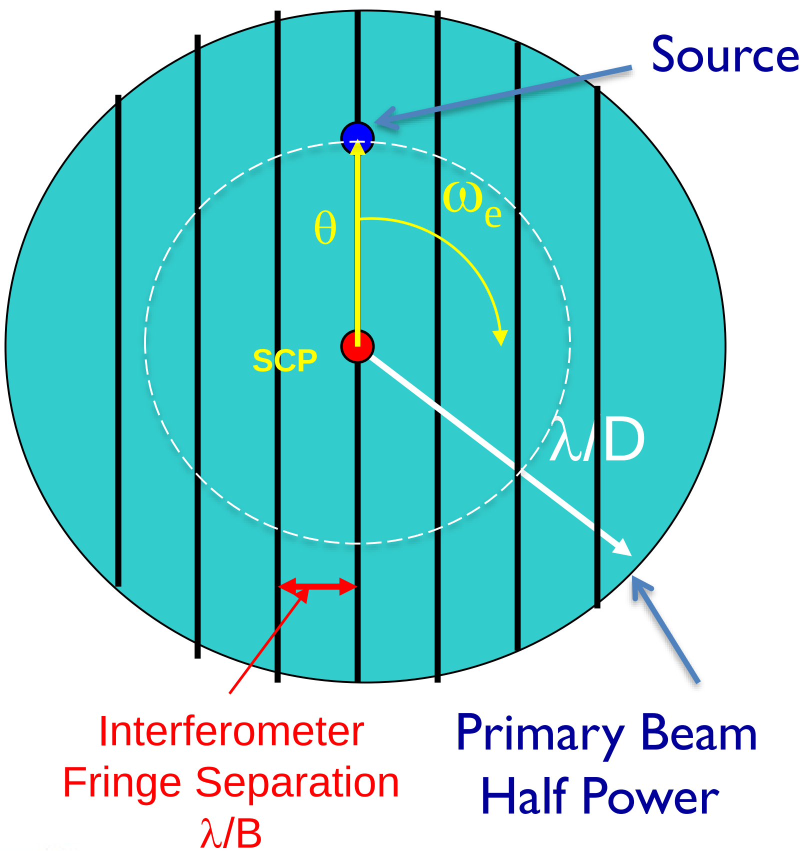
Maintaining phase stability requires much faster tracking than maintaining amplitude. The process is much easier if the phase adjustment can be done independently of delay.

Time Averaging Loss

- We can track a moving source, continuously adjusting the delay and phase to move the fringe pattern with the source.
- This does two good things:
  - Slows down the data recording needs
  - Prevents bandwidth delay losses.
- But, you cannot increase the time averaging indefinitely.
- The reason is that the fringe tracking mechanism is correct for only one point in the sky. All others have a different rate.
- So while you can reduce the fringe rate to zero for any given place, all other directions retain a differential rate.
- The limit to time averaging set by this differential.

Time-Smearing Loss Timescale

Simple derivation of fringe period, from observation at the SCP.



- Turquoise area is antenna primary beam on the sky – radius = λ/D
- Interferometer coherence pattern has spacing = λ/B
- Sources in sky rotate about NCP at angular rate:

  $\omega_e = 7.3\mathrm{x}10^{-5}$ rad/sec.

- Minimum time taken for a source to move by λ/B at angular distance θ is:

$$t = \frac{\lambda}{B}\frac{1}{\omega_E \theta}$$

- For a source at the beam half power, θ=λ/D, so at that radius:

$$t = \frac{D}{B}\frac{1}{\omega_E}$$

- For the VLA in A configuration, t~10 seconds.

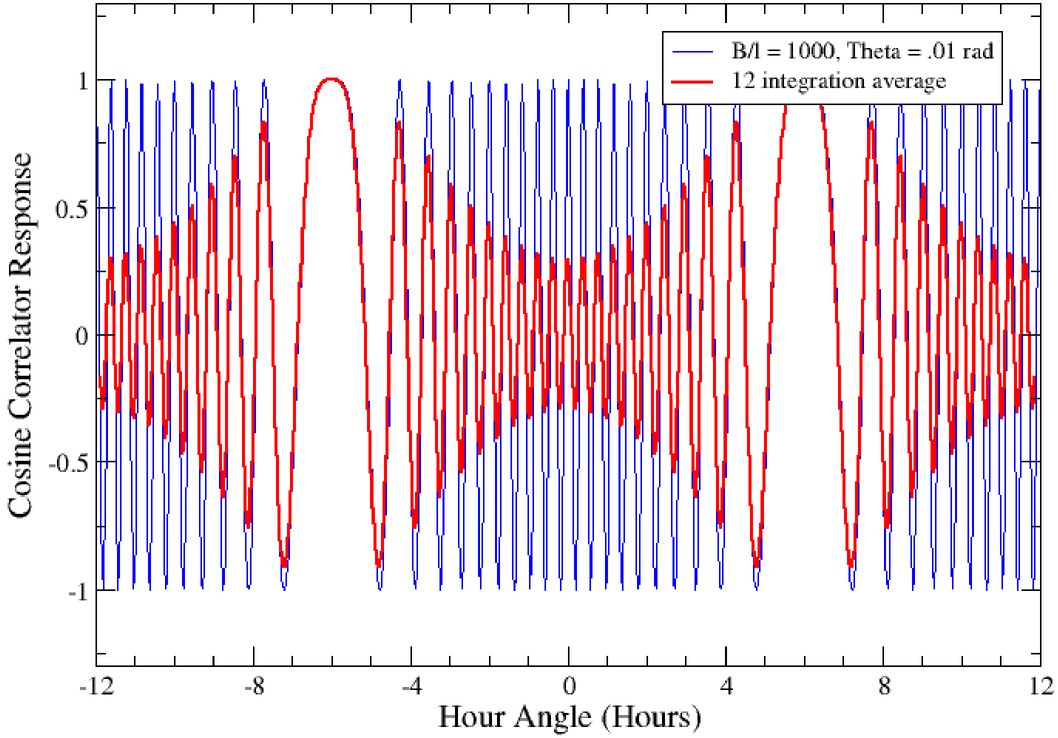
Illustrating Time Averaging Loss

- An object located away from the fringe tracking center moves through the fringe pattern as the earth rotates.
- It makes one cycle around in 24 hours.
- If we average the correlation products for too long a period, a loss in fringe amplitude will result.

Illustrating time average loss.

Blue trace: the fringe amplitude with no averaging.

Red trace: Amplitude after averaging for 12 'samples'.



Time-Averaging Loss

- So, what kind of time-scales are we talking about now?
- How long can you integrate before the differential motion destroys the fringe amplitude?
- **Case A:** A 25-meter parabaloid, and 35-km baseline:
  - $t = D/(B\omega_e) = 10$ seconds. (independent of observing frequency).
- **Case B**: Whole Hemisphere for a 35-km baseline:
  - $t = \lambda/(B\omega_e)$ sec = 83 msec at 21 cm.
- Averaging for durations longer than these will cause severe attenuation of the visibility amplitudes.
- To prevent 'delay losses', your averaging time must be much less than this.
  - Averaging time 1/10 of this value normally sufficient to prevent time loss.